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A METHOD FOR THE APPROXIMATE SOLUTION OF A TWO-PHASE STEFAN PROBLEM WITH REVERSE MOTION OF THE FRONT

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Determination of the trajectory of a phase transition front moving in a forward or reverse direction is reduced to the solution of an ordinary differential equation. A numerical check of the results shows the method to be highly accurate.

In the design of various apparata and structures, for example, wells in regions containing frozen rocks, the operation of which leads to a change in the aggregate state of the material in the surrounding medium, one is obliged to make numerous calculations of the motion of a phase transition boundary. Use of difference methods [1-3] for these purposes leads to the expenditure of a large amount of computer time, particularly in the case in which the process involves an infinite region. In this situation expenditures of computer time increases most perceptibly when solving problems involving a reverse front owing to the fact that the boundary of the computational domain must be moved especially far away. Reduction of an infinite domain to a finite one through a change of coordinates, for example, through use of the method indicated in [4], does not in practice decrease the volume of calculations. Moreover, as computational practice shows, difference methods cease to be suitable when the temperature of the initial phase is considerably below or above the temperature of the transition phase and the development of the process proceeds at extremely slow rates. Under these circumstances the role of approximate methods in carrying out engineering calculations is enhanced, particularly methods based on L. S. Leibenzon's integral formulation of the problem [5, 6]. This formulation, when used with suitable approximations of temperature profiles, makes it possible to obtain acceptable accuracy in determining the dynamics of the front of phase transitions and, in the first place, is interesting for practical applications. The version of the integral balance method presented in [6] is more effective, in this respect, than that given in [5] since in it terms not specified by the boundary conditions were excluded. This exclusion was effected in [6] by applying an operation of double integration; however, as shown in [7], the same result can be obtained by the use of Green's transformation. This modified version of the integral balance method, when applied to a one-phase problem, guarantees high accuracy in replacing the true temperature distribution by a quasistationary one, even for large Stefan numbers [7]. Obviously, this conclusion can also be carried over to the case of the two-phase problem since in the thermal balance integral the contributions from each of the phases are taken into account independently of one another.

In what follows, a modified integral balance method is developed for the case in which the motion of the phase front commences after a preliminary initial heating of the region and also when its forward motion changes to a reverse motion after thermal action ceases. In all these cases the trajectory of the front is described by a first order ordinary differential equation.

1. First of all, we obtain the differential equation for the case of an exterior single-phase Stefan problem with convective heat exchange at the moving boundary. After dimensionalization, this problem may be reduced to solving the heat-conduction equation in the region $1 \leqslant \xi \leqslant \eta$:

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$$\Delta\theta_1 = \frac{\partial\theta_1}{\partial\tau}; \quad \Delta\theta = \frac{1}{A(\xi)}\frac{\partial}{\partial\xi}A(\xi)\frac{\partial\theta}{\partial\xi} \tag{1}$$

with the boundary conditions

$$\xi=\eta: \quad \theta_1=0; \quad -K_1A(\eta)\left(\frac{\partial\theta_1}{\partial\xi}\right)_\eta = \dot{\eta}A(\eta)+K_2q_\eta(\tau); \tag{2}$$

$$\xi=1: \quad -A(1)\frac{\partial\theta_1}{\partial\xi}+h\theta_1=h\psi(\tau); \tag{3}$$

$$\tau=0: \quad \eta(0)=1. \tag{4}$$

Here $A(\xi) = \xi^k$, where k = 0, 1, 2 for plane-parallel, axial, and central symmetry, respectively; K_1 and K_2 are constant Stefan parameters; $q_\eta(\tau)$ is the heat output, which can depend both on η as well as on τ; h is a convective heat-exchange parameter (Biot number); $\psi(\tau)$ is the relative temperature of the medium. In the successive results allowance is made for the possibility of having $h = \infty$, in which case relation (3) becomes a boundary condition of the first kind and $\psi(\tau)$ then denotes temperature of the wall itself.

Multiplying differential equation (1) by the function $A(\xi)u(\xi)$

$$u(\xi) = \frac{1}{h}\left[1+h\int_1^\xi \frac{d\xi}{A(\xi)}\right], \tag{5}$$

we integrate the result from $\xi = 1$ to $\xi = \eta$. After simplifications, we then obtain the equation

$$\frac{d}{d\tau}\int_1^\eta u(\xi)\theta_1A(\xi)\,d\xi = u(\eta)A(\eta)\left(\frac{\partial\theta_1}{\partial\xi}\right)_\eta + \psi(\tau).$$

The first term on the right is eliminated through use of condition (2_2) and the latter equation is then rewritten in the form of the differential equation

$$\frac{d\tau}{d\eta} = \frac{\dfrac{d\psi_1}{d\eta}+\omega u(\eta)A(\eta)}{\psi(\tau)-\varepsilon u(\eta)q_\eta(\tau)}, \tag{6}$$

where

$$\varphi_1 = \int_1^\eta u(\xi)\theta_1A(\xi)\,d\xi; \quad \varepsilon = K_2/K_1; \quad \omega = 1/K_1.$$

If the dimensionless temperature of the medium behind the perturbed wall is constant, we can then always arrange to have it equal to one, $\psi(\tau) = 1$, through an appropriate normalization. Replacing θ_1 on the right side of Eq. (6) by the stationary profile

$$\theta_1 = \theta_s; \quad \theta_s = 1-\frac{u(\xi)}{u(\eta)}, \tag{7}$$

we obtain

$$\varphi_1 = \int_1^\eta u(\xi)\left[1-\frac{u(\xi)}{u(\eta)}\right]A(\xi)\,d\xi = J_1. \tag{8}$$

Equation (6), with φ_1 replaced by J_1, can then be solved as an ordinary differential equation with the initial condition $\tau = 0$ for $\eta = 1$.

If $\psi(\tau)$ varies with time, θ_1 is then replaced by the function $\psi(\tau)\theta_s(\xi, \eta)$, and then $\varphi_1 = \psi(\tau)J_1$. In this case, obtaining a numerical solution becomes somewhat involved.

For the single-phase problem $\varepsilon = 0$ and the equation is easily integrated, resulting in the following relationship between τ and η:

$$\int_0^\tau \psi\,d\tau = \frac{1}{K_1}\int_1^\eta u(\xi)A(\xi)\,d\xi + \psi(\tau)J_1(\eta).$$

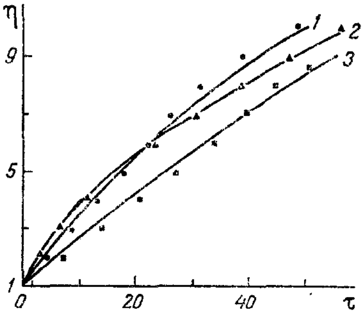

Fig. 1

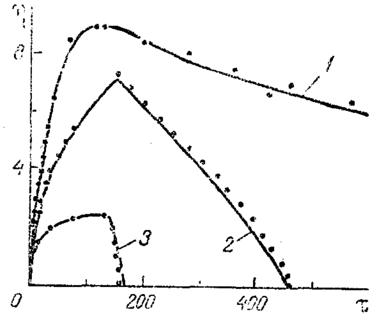

Fig. 2

Fig. 1. Phase transition front position η versus time τ for the plane-parallel geometry at different perturbation functions at the boundary. Curves, direct solution; points, solution from the differential equation with $K_1 = 1$: curve 1: $\psi(\tau) = 1.5 \sin (0.05\tau)$; curve 2: $\psi(\tau) = 0.3 \ln (1 + \tau)$; curve 3: $\psi(\tau) = 0.03\tau$.

Fig. 2. Phase transition front trajectory for planar wall (1), exterior of cylinder (2), and sphere (3) for unit temperature on the perturbing boundary in the initial period with subsequent cessation of thermal action. Curves, direct solution; points, solution from the differential equation with $K_1 = 0.95$, $K_2 = 0.272$, $\beta = 0.663$, $h = 5$.

It becomes especially simple for the plane-parallel case $A(\xi) = 1$ when a condition of the first kind is specified on the boundary. Then, letting $h = \infty$, we obtain from Eq. (5) the result $u(\xi) = \xi - 1$, which allows us, based on relations (7) and (8), to find

$$(\eta - 1)^2 = 2K_1 \int_0^\tau \psi \, d\tau \Big/ \left[1 + \frac{1}{3} K_1 \psi(\tau) \right].$$

Here $\psi(\tau)$ corresponds to the dimensionless temperature of the wall proper.

Figure 1 shows the results obtained when calculations made using this equation are compared with those using the method given in [3] for three different temperature functions on the planar wall: $\psi(\tau) = 1.5 \sin 0.05\tau$; $\psi(\tau) = 0.3 \ln (1 + \tau)$; $\psi(\tau) = 0.03\tau$, for a sufficiently large value of the parameter $K_1 = 1$. As can be seen, divergence of the results is minimal and completely acceptable for engineering purposes.

In the case of the single-phase problem, with the last approximation $\psi(\tau) = 1 - 0.2\tau$, a check with the numerical results given in [8] confirms its high accuracy up to $K_1 = 10$, considerably above the value of this parameter encountered in practical applications.

2. We consider now the situation, during the initial time interval $0 \leqslant \tau \leqslant \tau_1$, when the thermal action on the wall is defined by a condition of convective heat exchange with the medium with a constant temperature, which, for convenience, we can take equal to 1. In this time interval the boundary conditions are the same as those in relations (3) and (4) for $\psi(\tau) = 1$, while the motion of the front is described by Eq. (6). Next, starting from some time τ_1, thermal action on the wall ceases and in the following time interval the boundary condition (3) is replaced by the following:

$$\xi = 1: \quad \frac{\partial \theta_1}{\partial \xi} = 0; \quad \tau > \tau_1. \tag{3a}$$

Using the ordinary heat-balance equation, we find

$$\left(A \frac{\partial \theta_1}{\partial \xi} \right)_\eta - \left(A \frac{\partial \theta_1}{\partial \xi} \right)_1 = \frac{d}{d\tau} \int_1^\eta \theta_1 A \, d\xi.$$

The first term on the left side of this equation may be expressed in terms of the second boundary oondition (relation (2)); the second term, in accordance with relation (3a), is equal to zero. Consequently, for $\tau > \tau_1$ we can give it the form

$$-\frac{1}{K_1}\dot{\eta}A(\eta)-\varepsilon q_\eta(\tau)=\frac{d}{d\tau}\int_1^\eta \theta_1 A d\xi \qquad (9)$$

and consider it as the energy equivalent of the differential condition (3a).

Using the resulting integral conditions on the boundary, we can, without noticeable error, replace the true temperature profile in the phase adjacent to the wall by an approximate profile. In order for it to be consistent with the profile used earlier for this purpose, namely, the profile (7), we take it in the form $\theta_1 = \psi(\tau)\theta_S$, differing from the latter only by the variable factor $\psi(\tau)$. From the compatibility condition it follows that $\psi(\tau_1) = 1$, while elsewhere the function $\psi(\tau)$ is unknown and remains to be determined.

Substitution of the given profile into Eq. (9) yields

$$-\frac{1}{K_1}A(\eta)\dot{\eta}-\varepsilon q_\eta(\tau)=\frac{d}{d\tau}\psi(\tau)J_0-\dot{\eta}\frac{d}{d\eta}\psi(\tau)J_0, \qquad (10)$$

where $J_0=\int_1^\eta\left[1-\frac{u(\xi)}{u(\eta)}\right]A(\xi)d\xi$ is a function depending only on η.

Integrating Eq. (10) with respect to the time, we obtain, upon noting that $\psi(\tau_1) = 1$,

$$-\omega\int_{\eta_1}^{\eta}A(\xi)d\xi-\varepsilon\int_{\tau_1}^{\tau}q_\eta(\tau)d\tau=\psi(\tau)J_0(\eta)-J_0(\eta_1).$$

This equation can be given the following compact form:

$$\psi(\tau)=1+[D(\eta_1)-D(\eta)]/J_0(\eta), \qquad (11)$$

where

$$D(\eta)=J_0(\eta)+\omega\int_1^\eta A(\xi)d\xi+\varepsilon\int_0^\tau q_\eta(\tau)d\tau.$$

If $q_\eta(\tau)$ depends not only on τ but also on η, the latter may then be regarded as a function of τ and may be determined through a numerical integration of the equation.

Noting that $d\varphi_1/d\eta = d/d\eta(\psi J_1) = d/d\eta(\psi J_0)J_1/J_0$ and that ψJ_0 may be found from Eq. (10), we obtain the following after substituting it into the right side of equation (6) and transforming the result:

$$\frac{d\tau}{d\eta}=\frac{\psi C(\eta)+\omega A(\eta)B(\eta)}{\psi-\varepsilon B(\eta)q_\eta(\tau)}, \qquad (12)$$

where

$$B(\eta)=u(\eta)-\frac{J_1(\eta)}{J_0(\eta)}\geqslant 0;\quad C(\eta)=J_0\frac{d}{d\eta}\frac{J_1}{J_0}.$$

Together with Eq. (11), this differential equation allows us to determine the dependence of τ on η in the time interval $\tau \geqslant \tau_1$, following which the function $\psi(\tau)$ can be found from Eq. (11).

The quality of the approximate solution of the given problem based on the differential equation (12) is considered below along with an approximation for the output of heat from the front into the depth of the infinite region.

3. In order to generalize the method to the infinite region $\xi \geqslant \eta$, initially occupied by some one phase, it is necessary to find the amount of thermal flow on the boundary for a change in the aggregate state and to substitute it into the differential equation (6) for a forward motion of the front, or into Eq. (12) for frontal motion in the reverse direction. It is obvious that this flow is determined by the gradient of the dimensionless temperature θ_2, which satisfies the heat conduction equation

$$\Delta\theta_2 = \beta \frac{\partial\theta_2}{\partial\tau}; \quad \eta \leqslant \xi < \infty \tag{13}$$

with boundary conditions

$$\xi = \eta: \quad \theta_2 = 0; \quad \xi = \infty: \quad \theta_2 = -1; \quad \tau = 0: \quad \theta_2 = \theta^0(\xi). \tag{14}$$

For an exact solution of the problem the condition on the moving boundary is supplemented by the heat balance (second condition in relations (2)), which includes the desired thermal flow. Moreover, $\theta^0(\xi)$ is the temperature distribution in the region $\xi \geqslant 1$ at the start of melting, which, for example, is stipulated by preliminary initial heating in the presence of thermal insulation on the perturbed wall $\xi = 1$. In the absence of thermal insulation (for a boundary condition of the first kind) melting takes place instantaneously without preliminary initial heating of the region and then $\theta^0(\xi) = -1$. In this particular case the exact temperature profile can be replaced by an approximate one, thereby solving the boundary-value problem (13), (14) under the assumption that the left boundary $\xi = \eta$ is fixed. Then, for the plane-parallel and centrally symmetric cases the temperature distribution in the region $\xi \geqslant \eta$ will be given by the expressions

$$\begin{gathered} \theta_2(\xi, \tau) = \operatorname{erfc} \frac{\xi - \eta}{2\sqrt{\tau/\beta}} - 1, \\ \theta_2(\xi, \tau) = \frac{\eta}{\xi} \operatorname{erfc} \frac{\xi - \eta}{2\sqrt{\tau/\beta}} - 1. \end{gathered} \tag{15}$$

From this we then obtain, for the thermal flow on the boundary $\xi = \eta$, the following expression:

$$q_\eta(\tau) = \sqrt{\frac{\beta}{\pi\tau}}; \qquad q_\eta(\tau) = \eta + \eta^2 \sqrt{\frac{\beta}{\pi\tau}} \tag{16}$$

for the plane-parallel and spherical heat distributions, respectively. For the plane-parallel case, under the same assumptions, the heat output is given to high accuracy by É. B. Chekalyuk's formula [9]:

$$q_\eta(\tau) = \ln^{-1}\left[1 + \sqrt{\frac{\pi\tau}{\beta\eta^2}}\right]. \tag{17}$$

During the forward motion of the front Eqs. (16) and (17) yield somewhat lowered values for the heat output at the moving boundary when compared with the exact solution; this can be shown with the aid of the comparison theorem from [10].

To improve these approximations we make use of the thermal balance equation for the region occupied by the initial phase:

$$\bar{q}_\eta(\tau) = \beta\dot{\eta}A(\eta) + \beta \frac{d}{d\tau} \int_\eta^\infty [\theta_2(\xi, \tau) + 1]\, A(\xi)\, d\xi. \tag{18}$$

This equation may be obtained from the heat-conduction equation (13) by multiplying it by $A(\xi)d\xi$ and then integrating over the limits of the assigned region. Upon substituting into Eq. (18) the approximations for the temperature profile obtained earlier, we obtain the following expression for the heat output at the moving boundary:

$$\bar{q}_\eta(\tau) \simeq \beta\dot{\eta}A(\eta) + q_\eta(\tau). \tag{19}$$

As can be seen, this new approximation differs from those presented earlier only by the addition of the term $\beta\dot{\eta}A(\eta)$. Use of this approximation amounts to replacing the parameter ω in all the expressions derived earlier by the value $\omega = (1 + \beta K_2)/K_1$; at the same time, the expressions for $q_\eta(\tau)$ are understood to be those indicated in equations (16) and (17). Otherwise, the differential equation (6) or (12) and the expressions appearing in it, symbol for symbol, stay the same.

We remark that expressions of type (19) for plane-parallel flow were also used in [6].

In accordance with the expressions given, the heat output (16), (17), (19), at the instant that melting commences and from which instant time τ is reckoned, is infinite; this is compatible only with a boundary condition of the first kind at the perturbed wall. In the event of preliminary initial heating of the region up to the start of melting, the expression to be used for the heat output is $q_\eta(\tau + \tau_0)$, where τ_0 is determined from a condition of compatibility of the heat flows specified by this expression and the initial temperature distribution

$$q_{\eta=1}(\tau_0) = -\left(A(\xi)\frac{\partial\theta^0}{\partial\xi}\right)_{\xi=1}.$$

If the preliminary initial heating is stipulated by a thermally insulated wall with heat transfer coefficient k, we can readily show that

$$-\left(A(\xi)\frac{\partial\theta^0}{\partial\xi}\right)_{\xi=1} = h_0\frac{T_0 - T_i}{T_m - T_i} = \frac{h}{\varepsilon},$$

where $h_0 = kr_0/\lambda_2$; $h = kr_0/\lambda_1$; $\varepsilon = K_2/K_1$; r_0 is a characteristic scale, for example, the well radius; λ_i is the phase thermal conductivity coefficient, with subscript 2 corresponding to the initial phase.

By the same token, we obtain the following expressions for the compatibility time τ_0 based on the expressions adopted for the heat output:

$$\sqrt{\tau_0} = \frac{\varepsilon}{h}\sqrt{\frac{\beta}{\pi}}; \quad \sqrt{\tau_0} = \frac{\varepsilon}{h - \varepsilon}\sqrt{\frac{\beta}{\pi}}$$

for plane-parallel and spherical geometries, and

$$\sqrt{\tau_0} = \sqrt{\frac{\beta}{\pi}}\left(\exp\frac{\varepsilon}{h} - 1\right)$$

for cylindrical geometry.

Figure 2 shows results of calculations made using our method in comparison with the direct solution using the method in [3] for a planar wall, the exterior of a cylinder, and a sphere. The following values of the dimensionless parameters were used in our calculations: $K_1 = 0.95$; $K_2 = 0.272$; $\beta = 0.667$; $h = 5$. We remark that for the time τ_1 of thermal action cessation we used a somewhat smaller value for the planar wall than we did for the cylinder and sphere in order to accommodate results of a comparative calculation on one graph.

In a comparison of the front trajectories for the various kinds of symmetry, notice should be taken of the abrupt slowing of the rates of motion and the steepness of the reverse portion of the trajectory in the case of the sphere. This is due to the fact that the spherical front is bounded by a specific coordinate $\eta_{st} = 1 + \varepsilon/\varepsilon\ h/1 + h$, whose value is easily obtained by examining the denominator of equation (6) following a disclosure of the functions appearing in it in connection with a given concrete case.

As is evident from Fig. 2, our method gives roughly the same results as the method involving a direct solution of the Stefan problem using finite differences [3]. Divergence in the calculated results is minimal and not greater than 5%.

The method presented here was used recently over a period of several years to solve various problems connected with the thawing of frozen rocks and their refreezing; among the problems considered was that of the ablation of rocks during the channeling of wells, in which the method invariably proved its effectiveness. This furnishes a basis for recommending it for broader usage.

NOTATION

ξ, τ, dimensionless distance and time; η, phase transition front coordinate; $\dot\eta = d\eta/d\tau$; θ_1, θ_2, relative temperatures of the first and second phases; β, thermal diffusivity ratio for the first and second phases; $\Delta\theta$, Laplacian.

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COEFFICIENT INVERSE HEAT-CONDUCTION PROBLEM

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The computational algorithm and the results are given for the solution of the inverse problem of determining the total set of coefficients of the inhomogeneous quasilinear heat-conduction equation.

Recently, nonsteady experimental—computational methods based on the solution of the coefficient (in the terminology of [1]) inverse heat-conduction problems (IHP) have been sufficiently widely used to determine the thermophysical characteristics of various structural and heat-protective materials. Expansion of the range of practical application of such methods is directly associated with the development of effective computational algorithms for the solution of nonlinear multiparameter inverse problems in which a whole set of unknown characteristics is determined from the data of a single nonsteady experiment. This type of algorithm may ensure maximum information retrieval from thermophysical experiments.

Consider a one-dimensional heat-transfer process with a mathematical model in the form of a boundary problem for the quasilinear inhomogeneous heat-conduction equation

$$C(T)\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right)+S(T),\ x,\ \tau\in Q=(0,\ b)\times(0,\ \tau], \quad (1)$$

$$T(x,\ 0)=T_0(x),\ x\in[0,\ b], \quad (2)$$

$$\gamma_1\lambda(T(0,\ \tau))\frac{\partial T(0,\ \tau)}{\partial x}+\mu_1 T(0,\ \tau)=g_1(\tau),\ \tau\in(0,\ \tau_m], \quad (3)$$

$$\gamma_2\lambda(T(b,\ \tau))\frac{\partial T(b,\ \tau)}{\partial x}+\mu_2 T(b,\ \tau)=g_2(\tau),\ \tau\in(0,\ \tau_m], \quad (4)$$

where $T_0(x)$, $g_1(\tau)$, $g_2(\tau)$ are known functions; b, τ_m, μ_1, μ_2, γ_1, γ_2 are specified numbers.

Suppose that thermosensors are placed at some number (N + 2) of points in the interval [0, b] with coordinates $x = X_i$, $i = \overline{1, N}$, $0 = X_0 < X_1 < \ldots < X_N < X_{N+1} = b$, and dynamic temperature measurements are undertaken

$$T^{\exp}(X_i,\ \tau)=f_i(\tau),\ i=\overline{0,\ N+1}. \quad (5)$$

It is assumed here that, if a boundary condition of the first kind is imposed at any boundary, the functions $g_j(\tau)$, j = 1, 2 in Eqs. (3) and (4) are formed on the basis of the data of the corresponding measurements $g_1(\tau) = f_0(\tau)$, $g_2(\tau) = f_{N+1}(\tau)$. Depending on a priori information on the characteristics C(T), λ(T), and S(T), different formulations of the coefficient IHP are possible: the derivation of any one characteristic or some set of characteristics simultane-

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